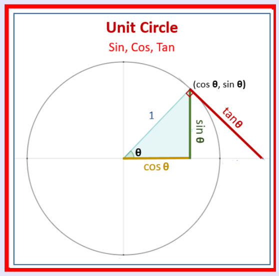
Math 241

Winter 2023

Lecture 13



Feb 19-8:47 AM

Solving Trig. Equations:

Recall from algebra, we can solve

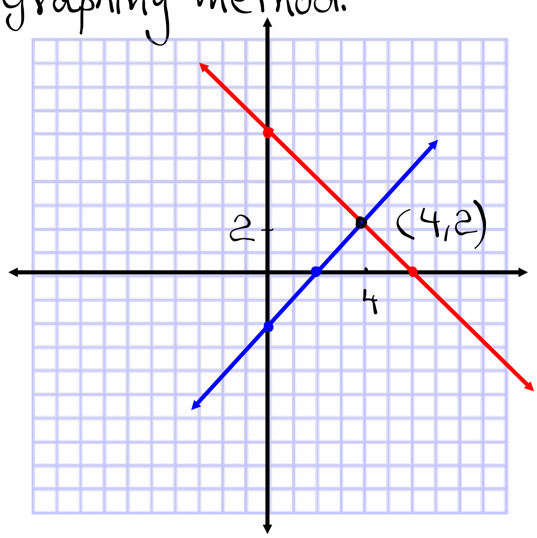
$$\begin{cases} x + y = 6 \\ x - y = 2 \end{cases}$$ by graphing method.

$x + y = 6$

x	y
0	6
6	0

$x - y = 2$

x	y
0	-2
2	0



Jan 25-7:03 AM

Solve

$$\begin{cases} 2x + 3y = 6 \\ y = \frac{-2}{3}x - 4 \end{cases} \Rightarrow \text{No Solution}$$

$\emptyset$

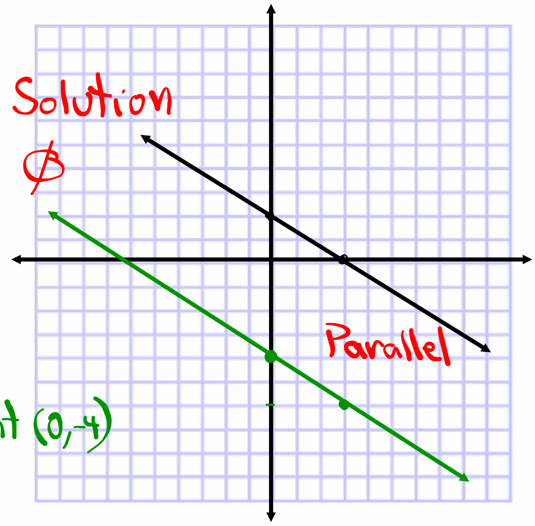
$2x + 3y = 6$

x	y
0	2
3	0

$y = \frac{-2}{3}x - 4$

$y = mx + b$

$m = \frac{-2}{3}$, Y-Int $(0, -4)$



Jan 25-7:06 AM

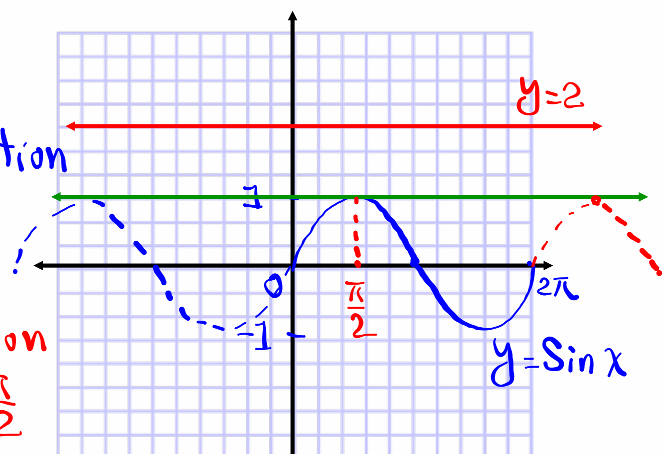
$$\begin{cases} y = \sin x \\ y = 2 \end{cases}$$

No Solution

$$\begin{cases} y = \sin x \\ y = 1 \end{cases}$$

Solution

$x = \frac{\pi}{2}$



we get infinite # of solution if we graph more periods.

General Solution

$x = \frac{\pi}{2} + n \cdot 2\pi$

$$\boxed{x = \frac{\pi}{2} + 2n\pi}$$

n is any integer.

Jan 25-7:10 AM

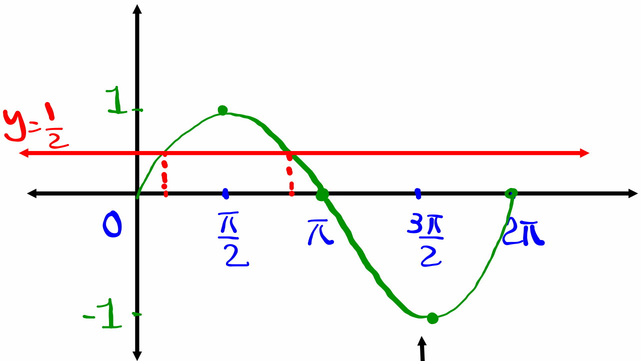
Solve $2\sin x - 1 = 0$

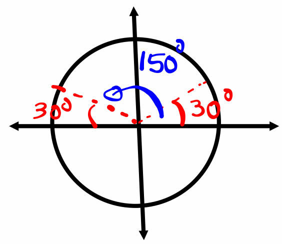
$2\sin x = 1$

$\sin x = \frac{1}{2}$

what is my R.A.?

$30^\circ, \frac{\pi}{6}$





QI $\boxed{x = \frac{\pi}{6} + 2n\pi}$

QII $x = \pi - \frac{\pi}{6} + 2n\pi$

$\boxed{x = \frac{5\pi}{6} + 2n\pi}$

General Solutions

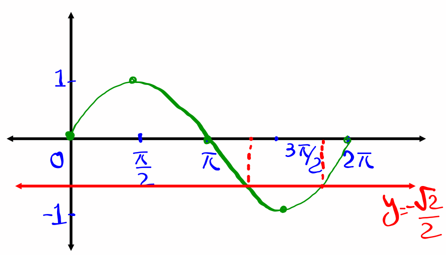
Jan 25-7:15 AM

Solve

$2\sin x + \sqrt{2} = 0$

$2\sin x = -\sqrt{2}$

$\sin x = \frac{-\sqrt{2}}{2}$



what is the Ref. Angle?

$45^\circ, \frac{\pi}{4}$

QIII

$x = \pi + RA + 2n\pi$

$= \pi + \frac{\pi}{4} + 2n\pi$

$\boxed{x = \frac{5\pi}{4} + 2n\pi}$

QIV

$x = 2\pi - RA + 2n\pi$

$x = 2\pi - \pi/4 + 2n\pi$

$\boxed{x = \frac{7\pi}{4} + 2n\pi}$

General Solutions

$5\frac{\pi}{4} = 225^\circ$

$\frac{7\pi}{4} = 315^\circ$

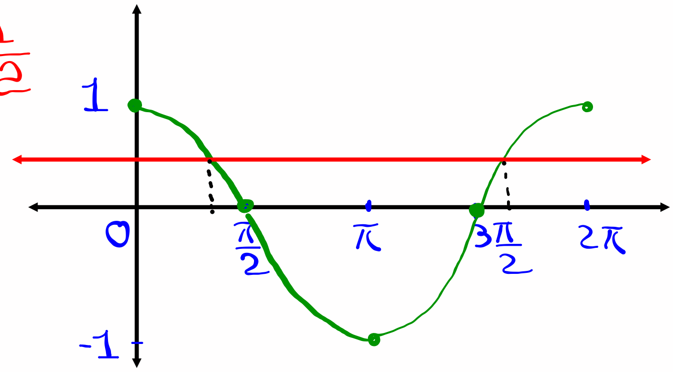
$\boxed{x = 225^\circ + 360^\circ \cdot n}$

$\boxed{x = 315^\circ + 360^\circ \cdot n}$

Jan 25-7:22 AM

Solve $2\cos x - 1 = 0$

$\cos x = \frac{1}{2}$



What is the RA?

$60^\circ, \frac{\pi}{3}$

QI

$x = RA + 2n\pi$

$\boxed{x = \frac{\pi}{3} + 2n\pi}$

$x = 60^\circ + 360^\circ \cdot n$

QIV

$x = 2\pi - RA + 2n\pi$

$= 2\pi - \frac{\pi}{3} + 2n\pi$

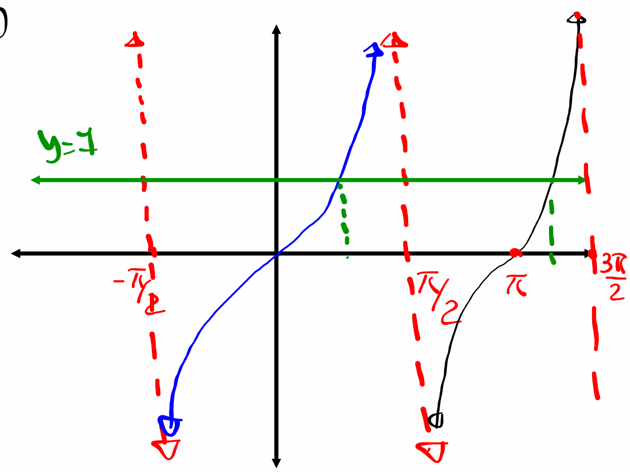
$\boxed{x = \frac{5\pi}{3} + 2n\pi}$ $x = 300^\circ + 360^\circ \cdot n$

Jan 25-7:30 AM

Solve $\tan x - 1 = 0$

$\tan x = 1$

R.A. $\rightarrow \frac{\pi}{4}, 45^\circ$



$x = R.A. + n\pi$

$\boxed{x = \frac{\pi}{4} + n\pi}$

Jan 25-7:38 AM

Solve $\tan^2 x - 1 = 0$

What is the R.A.?

$\frac{\pi}{4}$, $45°$

$$(\tan x - 1)(\tan x + 1) = 0$$

QI

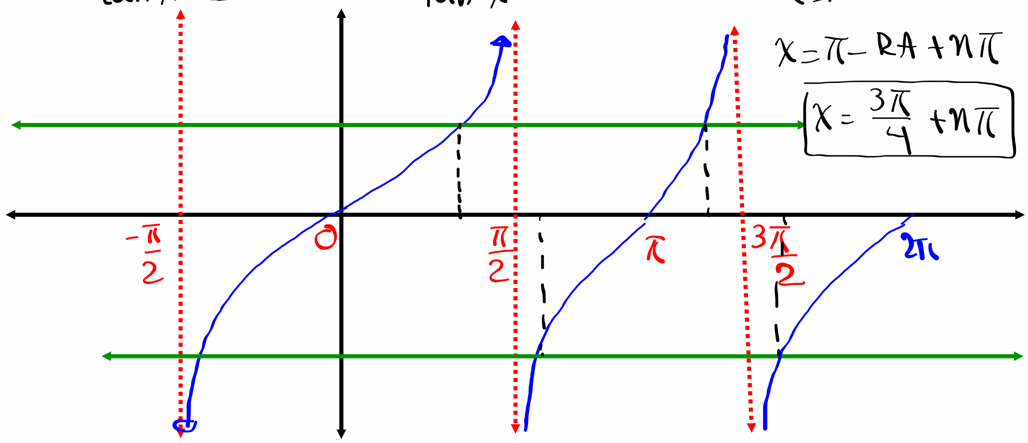
$\tan x - 1 = 0$ $\qquad$ $\tan x + 1 = 0$ $\qquad$ $\boxed{x = \frac{\pi}{4} + n\pi}$

$\tan x = 1$ $\qquad$ $\tan x = -1$

QII

$x = \pi - RA + n\pi$

$\boxed{x = \frac{3\pi}{4} + n\pi}$



Jan 25-7:44 AM

Arguement of Trig. function $\Rightarrow$ Arg.

QI $\quad$ Arg. = R.A. + n · period

QII $\quad$ Arg. = π − R.A. + n · Period

QIII $\quad$ Arg. = π + R.A. + n · Period

QIV $\quad$ Arg. = 2π − R.A. + n · Period

Then Solve for the Variable

Jan 25-7:51 AM

Solve $2\sin 2x - 1 = 0$

$2\sin 2x = 1$

$\sin 2x = \frac{1}{2}$

R.A. 30°

QI $\quad 2x = 30^\circ + n \cdot 360^\circ \rightarrow \boxed{x = 15^\circ + n \cdot 180^\circ}$

QII $\quad 2x = 180^\circ - 30^\circ + n \cdot 360^\circ$

$2x = 150^\circ + n \cdot 360^\circ \rightarrow \boxed{x = 75^\circ + n \cdot 180^\circ}$

Jan 25-7:54 AM

Solve $2\cos\frac{x}{2} + \sqrt{2} = 0$

$2\cos\frac{x}{2} = -\sqrt{2}$

$\cos\frac{x}{2} = -\frac{\sqrt{2}}{2}$

R.A. $\rightarrow 45^\circ$

QII $\quad$ Arg. $= 180^\circ -$ RA $+ n \cdot$ Period

$\frac{x}{2} = 180^\circ - 45^\circ + n \cdot 360^\circ$

$\frac{x}{2} = 135^\circ + n \cdot 360^\circ$

$\boxed{x = 270^\circ + n \cdot 720^\circ}$

QIII $\quad$ Arg. $= 180^\circ +$ R.A. $+ n \cdot$ Period

$\frac{x}{2} = 180^\circ + 45^\circ + n \cdot 360^\circ$

$\frac{x}{2} = 225^\circ + n \cdot 360^\circ$

$x = 450^\circ + n \cdot 720^\circ$

$x = 90^\circ + 360^\circ + n \cdot 720^\circ$

$x = 90^\circ + (2n+1) \cdot 360^\circ$

$360^\circ + n \cdot 720^\circ =$

$360^\circ + 2n \cdot 360^\circ =$

$(1 + 2n) 360^\circ$

Jan 25-7:59 AM

$\csc \frac{x}{3} = \frac{2\sqrt{3}}{3} \Rightarrow \sin \frac{x}{3} = \frac{3}{2\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\cancel{3}\sqrt{3}}{2 \cdot \cancel{3}} = \frac{\sqrt{3}}{2}$

Solve $\sin \frac{x}{3} = \frac{\sqrt{3}}{2}$

R.A. $\rightarrow \frac{\pi}{3}$

QI, QII

QI

Arg. = R.A. + n · Period

$\frac{x}{3} = \frac{\pi}{3} + n \cdot 2\pi$

$x = \pi + n \cdot 6\pi = (6n+1)\pi$

QII Arg. = π − R.A. + n · Period

$\frac{x}{3} = \pi - \frac{\pi}{3} + n \cdot 2\pi$

$x = 3\pi - \pi + n \cdot 6\pi = 2\pi + n \cdot 6\pi = (6n+2)\pi$

Jan 25-8:07 AM

Solve $\sin 2x - 1 = 0$ in the interval $[0, 2\pi)$.

$\sin 2x = 1$

R.A. $\frac{\pi}{2}$

Arg. = R.A. + n · 2π

$2x = \frac{\pi}{2} + n \cdot 2\pi$

$x = \frac{\pi}{4} + n \cdot \pi$

$n = 0 \rightarrow x = \frac{\pi}{4}$

$n = 1 \rightarrow x = \pi/4 + \pi$

$n = 2 \rightarrow \cancel{x = \frac{\pi}{4}} + 2\pi$

$\Rightarrow \frac{\pi}{4}, \frac{5\pi}{4}$

$\left\{ \frac{\pi}{4}, \frac{5\pi}{4} \right\}$

Jan 25-8:44 AM

$2\cos 3x - 1 = 0$ Find all Solutions in the interval $[0°, 360°)$

$\cos 3x = \frac{1}{2}$ ✓

R.A. $60°$

QI, QIV

QI

Arg. = R.A. + $n \cdot 360°$

$3x = 60° + n \cdot 360°$

$x = 20° + n \cdot 120°$

QIV

Arg = $360° -$ RA. $+ n \cdot 360°$

$3x = 360° - 60° + n \cdot 360°$

$3x = 300° + n \cdot 360°$

$x = 100° + n \cdot 120°$

$n=0 \rightarrow x = 100°$

$n=1 \rightarrow x = 220°$

$n=2 \rightarrow x = 340°$

$n=0$ $x=20°$

$n=1$ $x=140°$

$n=2$ $x=260°$

~~$n=3$ $x=380°$~~

$\{20°, 100°, 140°, 220°, 260°, 340°\}$

Let's check

$x = 140°$

$\cos(3 \cdot 140°) = \cos 420° = \frac{1}{2}$ ✓

Jan 25-8:48 AM

$(\tan 2x - 1)(\tan 2x + \sqrt{3}) = 0$, Solve in the interval $[0, 2\pi)$

$\tan 2x - 1 = 0$

$\tan 2x = 1$

R.A. $\frac{\pi}{4}$

QI, QIII

QI: Arg = R.A. + $n \cdot \pi$

$2x = \frac{\pi}{4} + n \cdot \pi$

$x = \frac{\pi}{8} + \frac{n\pi}{2}$

QIII Arg. = π + R.A. + $n \cdot \pi$

$2x = \pi + \frac{\pi}{4} + n \cdot \pi$

$2x = \frac{5\pi}{4} + n\pi$

$x = \frac{5\pi}{8} + \frac{n\pi}{2}$

$n=0 \rightarrow x = \frac{\pi}{8}$ ✓

$n=1 \rightarrow x = \frac{\pi}{8} + \frac{\pi}{2} = \frac{5\pi}{8}$ ✓

$n=2 \rightarrow x = \frac{\pi}{8} + \frac{2\pi}{2} = \frac{9\pi}{8}$ ✓

$n=3 \rightarrow x = \frac{\pi}{8} + \frac{3\pi}{2} = \frac{13\pi}{8}$ ✓

~~$n=4 \rightarrow x = \frac{\pi}{8} + \frac{4\pi}{2}$~~

$n=0 \rightarrow x = \frac{5\pi}{8}$ Repeated

$n=1 \rightarrow x = \frac{9\pi}{8}$ Repeated

$n=2 \rightarrow x = \frac{13\pi}{8}$ "

~~$n=3 \rightarrow x = \frac{17\pi}{8}$~~

Jan 25-8:56 AM

$\tan 2x + \sqrt{3} = 0$

$\tan 2x = -\sqrt{3}$

R.A. $\pi/3$

QII , QIV

QII:

Arg. = π − R.A. + n · Period

$2x = \pi - \frac{\pi}{3} + n \cdot \pi$

$2x = \frac{2\pi}{3} + n\pi$

$x = \frac{\pi}{3} + \frac{n\pi}{2}$

$n=0 \rightarrow x = \frac{\pi}{3}$

$n=1 \rightarrow x = \frac{5\pi}{6}$

$n=2 \rightarrow x = \frac{4\pi}{3}$

$n=3 \rightarrow x = \frac{11\pi}{6}$

$n=4$ ✗

QIV

Arg. = 2π − RA + n · Period

$2x = 2\pi - \frac{\pi}{3} + n \cdot \pi$

$2x = \frac{5\pi}{3} + n \cdot \pi$

$x = \frac{5\pi}{6} + n \cdot \pi/2$

$n=0 \rightarrow x$

$n=1 \rightarrow x$

$n=2 \rightarrow x$

$n=3 \rightarrow x$

Jan 25-9:07 AM

Solve $\sin \frac{1}{2}\theta = -\cos \frac{1}{2}\theta$

over $[0°, 360°)$

Divide by $\cos\frac{1}{2}\theta$ both sides

$\frac{\sin \frac{1}{2}\theta}{\cos \frac{1}{2}\theta} = -1 \Rightarrow \tan \frac{1}{2}\theta = -1$

R.A. → 45°

QII , QIV

QII

Arg. = 180° − R.A. + n · 180°

$\frac{1}{2}\theta = 180° - 45° + n \cdot 180°$

$\frac{1}{2}\theta = 135° + n \cdot 180° \rightarrow \theta = 270° + n \cdot 360°$

$n=0 \rightarrow \theta = 270°$

$n=1$ ~~→ θ = 270° + 360°~~

QIV

Arg. = 360° − R.A. + n · 180°

$\frac{1}{2}\theta = 360° - 45° + n \cdot 180°$

$\frac{1}{2}\theta = 315° + n \cdot 180°$ $\quad \theta = 630° + n \cdot 360°$

Check

$\sin\left(\frac{1}{2} \cdot 270°\right) = -\cos\left(\frac{1}{2} \cdot 270°\right)$ $\quad \{270°\}$

$\sin 135° = -\cos 135°$

$\frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2}$ ✓

Jan 25-9:14 AM

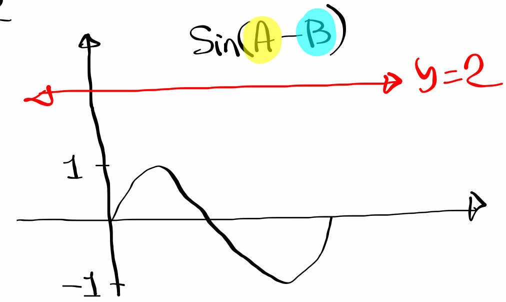
Solve $\sin 3\theta \cos\theta - \cos 3\theta \sin\theta = 2$

$\sin(3\theta - \theta) = 2$

used $\sin(A-B)$

$\sin 2\theta = 2$

$\varnothing$



Jan 25-9:22 AM

Solve

$\sin 6x - \sin 2x = 0$ in $[0°, 360°)$

$\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$

$\sin 6x - \sin 2x = 2\cos\frac{6x+2x}{2}\sin\frac{6x-2x}{2}$

$2\cos 4x \sin 2x = 0$

$\cos 4x \sin 2x = 0$

$\cos 4x = 0$ OR $\sin 2x = 0$

$90°, 270°$ $0°, 180°$

$4x = 90° + n\cdot 360°$ $2x = 0° + n\cdot 360°$

$x = 22.5° + n\cdot 90°$ $x = n\cdot 180°$

$4x = 270° + n\cdot 360°$ $2x = 180° + n\cdot 360°$

$x = 67.5° + n\cdot 90°$ $x = 90° + n\cdot 180°$

$\{0°, 22.5°, 67.5°, 90°, 112.5°, 157.5°, 180°, 270°, 202.5°, 247.5°, 292.5°, 337.5°\}$

Jan 25-9:26 AM

Solve $\sin^2 2\theta = 4 - 2\cos^2 2\theta$ on $[0°, 360°)$

Hint: use identities to write in terms of one trig. function.

$$\sin^2 x + \cos^2 x = 1$$

$$\cos^2 x = 1 - \sin^2 x$$

$$\sin^2 2\theta = 4 - 2(1 - \sin^2 2\theta)$$

$$\sin^2 2\theta = 4 - 2 + 2\sin^2 2\theta$$

$$\sin^2 2\theta - 2\sin^2 2\theta = 2$$

$$-\sin^2 2\theta = 2$$

$$\sin^2 2\theta = -2$$

$$(\text{Any Real \#})^2 \geq 0$$

$$\emptyset$$

Jan 25-9:38 AM

Solve $\csc^2 x = \cot x + 3$ in $[0°, 360°)$

Hint: Think of an identity that has $\csc^2 x$.

$$1 + \cot^2 x = \csc^2 x$$

$$1 + \cot^2 x = \cot x + 3$$

$$\cot^2 x - \cot x - 2 = 0$$

Can You factor this?

$$(\cot x + 1)(\cot x - 2) = 0$$

$\cot x + 1 = 0$ | $\cot x - 2 = 0$

$\cot x = -1$ | $\cot x = 2$

$\tan x = -1$ | $\tan x = \frac{1}{2}$

RA $45°$ | RA $\tan^{-1}(.5) \approx 27°$

QII, QIV | QI, QIII

QII

Arg. $= 180° - \text{RA}. + n \cdot 180°$

$x = 180° - 45° + n \cdot 180°$

$x = 135° + n \cdot 180°$

$n = 0 \to x = 135°$ ✓

$n = 1 \to x = 315°$ ✓

Arg. $= 360° - \text{RA} + n \cdot 180$

$x = 315° + n \cdot 180°$

$x = 27° + n \cdot 180°$

$n = 0 \to x = 27°$ ✓

$n = 1 \to x = 207°$

$x = 180° + 27° + n \cdot 180°$

$x = 207° + n \cdot 180°$

$$\{135°, 315°, 27°, 207°\}$$

Check $27°$:

$\csc^2 x = \cot x + 3$

$\csc^2 27° \overset{?}{=} \cot 27° + 3$

$4.852 \overset{?}{=} 4.963$ due to Rounding

Check $x = 315°$

$\csc^2 315° \overset{?}{=} \cot 315° + 3$

$2 \overset{?}{=} 2$ ✓

Jan 25-9:43 AM

Write $\sin 5x \sin 9x$ as a sum.

Use $\sin u \sin v = \frac{1}{2}\left[\cos(u-v) - \cos(u+v)\right]$

$$\sin 5x \sin 9x = \frac{1}{2}\left[\cos(5x-9x) - \cos(5x+9x)\right]$$

$$= \frac{1}{2}\left[\cos(-4x) - \cos(14x)\right]$$

$\cos(-\alpha) = \cos\alpha$

$$= \frac{1}{2}\left[\cos 4x - \cos 14x\right]$$

Write $\cos 3x \sin 7x$ as a sum.

$\cos u \sin v = \frac{1}{2}\left[\sin(u+v) - \sin(u-v)\right]$

$$\cos 3x \sin 7x = \frac{1}{2}\left[\sin 10x - \sin(-4x)\right]$$

$\sin(-\alpha) = -\sin\alpha$

$$= \frac{1}{2}\left[\sin 10x + \sin 4x\right]$$

Write $\sin 10x + \sin 4x$ as product

$\sin x + \sin y = 2 \sin \frac{x+y}{2} \cos \frac{x-y}{2}$

$$\sin 10x + \sin 4x = 2 \cdot \sin \frac{10x+4x}{2} \cos \frac{10x-4x}{2}$$

$$= 2 \sin 7x \cos 3x$$

Jan 25-10:33 AM

Write as product

$\cos 12x - \cos 6x$

$\cos x - \cos y = -2 \sin \frac{x+y}{2} \sin \frac{x-y}{2}$

$$\cos 12x - \cos 6x = \boxed{-2 \sin 9x \sin 3x}$$

Jan 25-10:43 AM

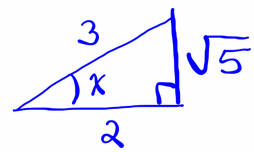
$\sec x = \frac{3}{2}$, $270^\circ < x < 360^\circ$ $\Rightarrow$ $135^\circ < \frac{x}{2} < 180^\circ$

QIV

Find $\cos\frac{x}{2}$

$\frac{x}{2}$ is in QII

Cos is $-$



$$\cos\frac{x}{2} = \pm\sqrt{\frac{1+\cos x}{2}}$$

$$= -\sqrt{\frac{1+\frac{2}{3}}{2}}$$

LCD=3

$$= -\sqrt{\frac{3+2}{6}} = -\sqrt{\frac{5\cdot 6}{6\cdot 6}}$$

$$= -\frac{\sqrt{30}}{\sqrt{36}} = \boxed{-\frac{\sqrt{30}}{6}}$$

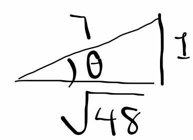
Jan 25-10:45 AM

$\sin\theta = \frac{1}{7}$, θ is in QII

$90^\circ < \theta < 180^\circ$

Find $\sin 2\theta$

$180^\circ < 2\theta < 360^\circ$



$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$= 2\cdot\frac{1}{7}\cdot\frac{-\sqrt{48}}{7} = \frac{-2\sqrt{48}}{49} = \frac{-2\cdot\sqrt{16}\sqrt{3}}{49}$$

$$= \boxed{\frac{-8\sqrt{3}}{49}}$$

Jan 25-10:51 AM

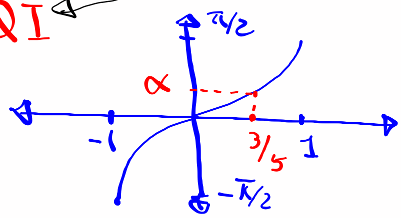
Find the exact Value

$$\tan\left(2\sin^{-1}\frac{3}{5}\right) = \tan 2\alpha$$

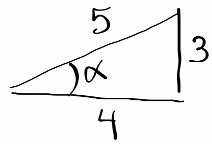
Let $\alpha = \sin^{-1}\frac{3}{5}$

$\sin\alpha = \frac{3}{5}$

QI



$$= \frac{2\tan\alpha}{1-\tan^2\alpha} = \frac{2\cdot\frac{3}{4}}{1-\left(\frac{3}{4}\right)^2}$$



$$= \frac{\frac{3}{2}}{1-\frac{9}{16}}$$

LCD=16

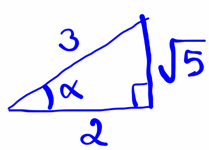
$$= \boxed{\frac{24}{7}}$$

Jan 25-10:55 AM

Find exact Value of

$$\tan\left(\frac{1}{2}\cos^{-1}\frac{2}{3}\right) = \tan\frac{\alpha}{2} = \frac{\sin\alpha}{1+\cos\alpha}$$

$\alpha = \cos^{-1}\frac{2}{3}$

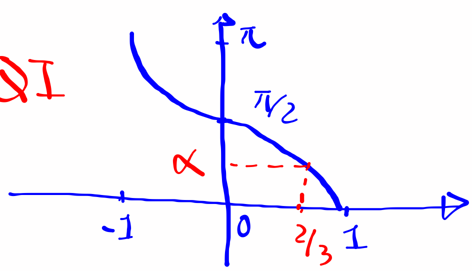


$$= \frac{\frac{\sqrt{5}}{3}}{1+2/3} = \boxed{\frac{\sqrt{5}}{5}}$$

$\cos\alpha = \frac{2}{3}$

LCD = 3

QI



Jan 25-10:59 AM

Verify

$$\frac{\sin 4x}{\sin x} = 4\cos x \cos 2x$$

$4x = 2 \cdot 2x$

$\sin 4x = \sin 2(2x)$

$= 2 \sin 2x \cos 2x$

$\sin 2x = 2 \sin x \cos x$

$$\frac{\sin 4x}{\sin x} = \frac{2 \sin 2x \cos 2x}{\sin x}$$

$$= \frac{2 \cdot 2 \cancel{\sin x} \cos x \cdot \cos 2x}{\cancel{\sin x}}$$

$$= \boxed{4 \cos x \cos 2x}$$

Jan 25-11:04 AM

Verify

$\cos^4 x - \sin^4 x = \cos 2x$

Hint: $A^2 - B^2 = (A+B)(A-B)$

$A^4 - B^4 = (A^2+B^2)(A^2-B^2)$

$$\cos^4 x - \sin^4 x = \underbrace{(\cos^2 x + \sin^2 x)}(\cos^2 x - \sin^2 x)$$

$$= 1 \cdot \cos 2x = \cos 2x$$

Verify $4(\sin^6 x + \cos^6 x) = 4 - 3\sin^2 2x$

Hint: $A^6 = (A^2)^3$

$A^3 + B^3 = (A+B)(A^2 - AB + B^2)$

$\sin^6 x + \cos^6 x =$

$(\sin^2 x)^3 + (\cos^2 x)^3 =$

$\big(\underbrace{\sin^2 x + \cos^2 x}_{1}\big)\big((\sin^2 x)^2 - \sin^2 x \cos^2 x + (\cos^2 x)^2\big) =$

$\sin^4 x - \sin^2 x(1 - \sin^2 x) + (1 - \sin^2 x)^2 =$

$\sin^4 x - \sin^2 x + \sin^4 x + 1 - 2\sin^2 x + \sin^4 x =$

$3\sin^4 x - 3\sin^2 x + 1 =$

$3\sin^2 x(\sin^2 x - 1) + 1 =$

$3\sin^2 x \cdot -\cos^2 x + 1 =$

$4(1 - 3\sin^2 x \cos^2 x) = 4 - 12 \sin^2 x \cos^2 x$

$= 4 - 3 \cdot 4 \sin^2 x \cos^2 x$

$= 4 - 3(2 \sin x \cos x)^2$

$= \boxed{4 - 3\sin^2 2x}$

Jan 25-11:09 AM